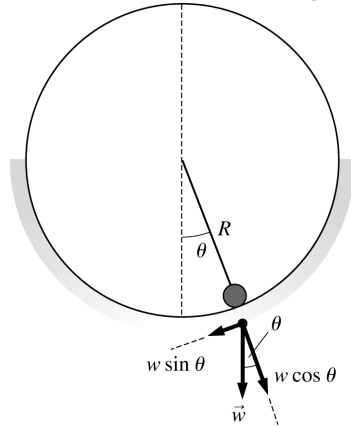


14.67. Model: Assume that $r_{\text{marble}} \ll R_{\text{hoop}}$ and that θ is a small angle.

Visualize:



Solve: The marble is like an object on an inclined plane. The net force on the marble in the tangential direction is

$$-w \sin \theta = ma = mR\alpha = mR \frac{d^2\theta}{dt^2} \Rightarrow -mg \sin \theta = mR \frac{d^2\theta}{dt^2}$$

where α is the angular acceleration. With the small-angle approximation $\sin \theta \approx \theta$, this becomes

$$\frac{d^2\theta}{dt^2} = -\frac{g}{R}\theta = -\omega^2\theta$$

This is the equation of motion of an object in simple harmonic motion with a period of

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{R}{g}}$$